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HEAT TRANSFER IN A MHD CHANNEL WITH UNIFORM WALL HEAT FLUX — EFFECTS OF HALL AND ION SLIP CURRENTS

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Abstract — The effects of the Hall and the ion slip currents on forced convective heat transfer in the thermal entrance region of a magnetohydrodynamic channel have been analysed by solving the energy equation as an eigenvalue problem. Both the generator and the accelerator modes are discussed. It is found that the heat-transfer rates are reduced due to these currents.

NOMENCLATURE

B_0, applied magnetic field;
E, electric field;
$Ec \cdot Pr$, $= (ku_m/c_p hq')(\eta' c_p/k)$ = heat generation parameter;
Ha, $= B_0 h(\sigma/\eta)^{1/2}$ = Hartmann number;
I, electric current density;
Nu, Nusselt number;
Pe, $= \rho u_m h c_p/k$;
T, temperature;
c_p, specific heat at constant pressure;
e, $= E/u_m B_0$ = dimensionless electric field;
h, half the height of the channel;
j, $= J/\sigma u_m B_0$ = dimensionless current density;
k, thermal conductivity;
q', rate of heat-transfer per unit area,
$$q/A' = -k\left(\frac{\partial T}{\partial z}\right)_{z=h};$$
u, v, dimensionless velocity components;
u_m, average velocity $= \int_1^1 u \, d\eta$.

Greek symbols

θ, $= \frac{T - T_0}{(hq'/k)}$ = non-dimensional temperature field;
ξ, $= x/Peh$ } dimensionless Cartesian coordinates;
ζ, $= y/h$
η, $= z/h$
σ, electrical conductivity;
η', dynamic viscosity;
ρ, density;
β_e, $= \omega_e \tau_e$ = Hall parameter;
β_i, $= f^2 \omega_i \tau_i$ = ion slip parameter;
ω, cyclotron frequency;
τ, collision time.

Subscripts

w, wall condition;
0, entrance condition, reference condition;
x, y, components in x, y direction;
e, electron;
i, ion.

INTRODUCTION

THE PHENOMENON of heat transfer in a magnetohydrodynamic (MHD) channel has been analysed by a number of authors due to its immediate application in many devices like MHD power generator, accelerator etc. These results are needed for the design of the duct wall and the cooling arrangements.

Using the Hartmann profile, the problem of forced convective heat transfer has been studied by Siegel [1], Gershuni and Zhukhovitskii [2], Michiyoshi and Matsumoto [3], Hwang *et al.* [4, 5] for the case of uniform wall heat flux and Nigam and Singh [6], Michiyoshi and Matsumoto [3] for uniform wall temperature. But the results are not very useful for the study of heat-transfer phenomenon in MHD devices where the working medium is a partially ionized gas (more commonly known as seeded plasma). This partially ionized gas is produced by mixing 1% of easily ionizable material (known as seed which is usually potassium or caesium) with some base gas. Due to the partial ionization and the presence of large magnetic fields, the electrical conductivity of the gas does not remain a scalar quantity. Transverse currents in the form of the Hall and the ion slip currents are produced in addition to the conducting current.

Eraslan and Eraslan [7, 8] have studied the effects of the Hall currents on the heat-transfer phenomenon for uniform wall temperature. The combined effects of the Hall and the ion slip currents on heat transfer have been studied by Mittal and Bhat [9] for uniform wall temperature and Javeri [10] for uniform wall heat flux.

The present note gives the effects of the Hall and the ion slip currents on the temperature profile and other heat-transfer coefficients in the thermal entrance region of a parallel plate MHD channel with uniform heat flux at the walls. The flow which is subsonic occurs under an externally applied electric field loading condition and a uniform transverse magnetic field. Approximating the gas by an electrically conducting viscous fluid with constant density, Javeri [11] and Mittal and Bhat [12] have already obtained, velocity distribution.

MATHEMATICAL FORMULATION

Consider a steady state fully developed laminar flow of an electrically conducting viscous fluid in a parallel plate channel of height $2h$, in the presence of a uniform magnetic field B_0, applied transversely to flow direction. The parallel plates are maintained at a uniform flux q'. The fluid enters the duct with a constant temperature T_0. Following Javeri [10], the equation of energy in the dimensionless form, for such a configuration, can be written as

$$u\frac{\partial\theta}{\partial\xi} = Pe^{-2}\frac{\partial^2\theta}{\partial\xi^2} + \frac{\partial^2\theta}{\partial\eta^2} + Ec\,Pr\left[\left(\frac{\partial u}{\partial\eta}\right)^2 + \left(\frac{\partial v}{\partial\eta}\right)^2\right] + Ec\,Pr\,Ha^2[(e_x + v)j_x + (e_y - u)j_y], \quad (1)$$

where

$$j_x = \frac{1}{\alpha_e^2 + \beta_e^2}[(e_x + v)\alpha_e - (e_y - u)\beta_e]$$

$$j_y = \frac{1}{\alpha_e^2 + \beta_e^2}[(e_x + v)\beta_e + (e_y - u)\alpha_e]$$

and $\alpha_e = 1 + \beta_e\beta_i$; β_e, β_i being the parameters for the Hall and the ion slip currents. θ denotes the dimensionless temperature and ξ and η are dimensionless coordinates in x and z directions given by

$$\xi = \frac{x}{Pe \cdot h}$$

$$\eta = \frac{z}{h}.$$

θ satisfies the following boundary conditions:

$$\theta = 0 \quad \text{at } \xi = 0, \qquad -1 \le \eta \le 1$$

$$\frac{\partial\theta}{\partial\eta} = 1 \quad \text{at } \eta = \pm 1, \qquad \xi > 0. \quad (2)$$

Solution procedure

Equation (1) is solved as an eigenvalue problem by writing

$$\theta(\xi,\eta) = \theta^+(\xi,\eta) + \theta_{fd}(\xi,\eta), \quad (3)$$

where θ_{fd} is the fully developed temperature distribution, and θ^+ is the temperature distribution in the absence of θ_{fd} in the thermal entrance region. Then θ_{fd} satisfies the equation

$$\frac{\partial^2\theta_{fd}}{\partial\eta^2} - u\frac{\partial\theta_{fd}}{\partial\xi} + Ec\cdot Pr\left[\left(\frac{\partial u}{\partial\eta}\right)^2 + \left(\frac{\partial v}{\partial\eta}\right)^2\right] + Ec\cdot Pr\cdot Ha^2\frac{\alpha_e}{\alpha_e^2 + \beta_e^2}[(e_x + v)^2 + (e_y - u)^2] = 0 \quad (4)$$

and θ^+ satisfies

$$u\frac{\partial\theta^+}{\partial\xi} = Pe^{-2}\frac{\partial^2\theta^+}{\partial\xi^2} + \frac{\partial^2\theta^+}{\partial\eta^2}. \quad (5)$$

The boundary conditions for θ_{fd} and θ^+ become

$$\frac{\partial\theta_{fd}}{\partial\eta} = 1 \quad \text{at } \eta = \pm 1, \quad \xi > 0$$

$$\frac{\partial\theta_{fd}}{\partial\xi} = A \quad (6)$$

(a constant to be determined later)

$$\frac{\partial\theta^+}{\partial\eta} = 0 \quad \text{at } \eta = \pm 1, \quad \xi > 0 \quad (7)$$

$$\theta^+ \to 0 \quad \text{as } \xi \to \infty. \quad (8)$$

Using the values of u and v as obtained earlier by Javeri [11] and Mittal and Bhat [12] in the form

$$u = A_{11}\cosh a\eta\cos b\eta - B_{11}\sinh a\eta\sin b\eta + C_{11}$$

$$v = B_{11}\cosh a\eta\cos b\eta + A_{11}\sinh a\eta\sin b\eta + D_{11}. \quad (9)$$

(The values of the constants $A_{11}, B_{11}, C_{11}, D_{11}, a$ and b are given in the Appendix.)

Javeri [9] has obtained the solution of equations (4) and (6) as

$$\theta_{fd} = \bar{A}\xi + F(\eta). \quad (10)$$

Here

$$F(\eta) = P_1\cosh 2a\eta + P_2\cos 2b\eta + P_3\sinh a\eta\sin b\eta + P_4\cosh a\eta\cos b\eta + P_5\eta^2 + P_6 \quad (11)$$

(The expressions for the constants $\bar{A}$ and P_1 to P_6 are given in the Appendix.)

Equation (5) with boundary conditions (7) and (8) is solved as an eigenvalue problem, using the method suggested by Millsaps and Pohlhausen [13] and Singh [14].

θ^+ satisfying condition (7) is written as

$$\theta^+ = \sum_{n=1,3,\ldots}^{\infty} W_n(\xi)\sin\frac{n\pi}{2}\eta \quad (12)$$

Substituting θ^+ in equation (5), multiplying both sides by $\sin(m\pi/2)\eta$ and integrating with respect to η from -1 to 1, it is easy to obtain

$$Pe^{-2}\frac{d^2W_n}{d\xi^2}-\Phi_1\frac{dW_n}{d\xi}-\left(\frac{n\pi}{2}\right)^2W_n=\sum_{\substack{m=1,3\\(m\neq n)}}^{\infty}\Phi_2\frac{dW_m}{d\xi}. \quad (13)$$

Here

$$\Phi_1=\int_{-1}^{1}u\sin^2\frac{n\pi}{2}\eta\,d\eta$$

and

$$\Phi_2=\int_{-1}^{1}u\sin\frac{n\pi}{2}\sin\frac{m\pi}{2}\eta\,d\eta.$$

(The integrated values of Φ_1 and Φ_2 are given in the Appendix.)

Taking the solution of equation (13) as

$$W_n(\xi)=A_n^{(p)}\exp(\lambda_p\xi),\ (p=1,3,5,\ldots,\infty) \quad (14)$$

and substituting these values of $W_n(\xi)$ in equation (13), the condition for determining the λ_ps is determined as

$$A_n^{(p)}f(\lambda_p,n)-\sum_{m=1,3}^{\infty}\Phi_2A_m^{(p)}\lambda_p=0, \quad (15)$$

where

$$f(\lambda_p,n)=Pe^{-2}\lambda_p^2-\lambda_p\Phi_1-\left(\frac{n\pi}{2}\right)^2.$$

The consistency condition for the set of equation (15) is

$$|A_{ij}(\lambda_p)|=0, \quad (16)$$

where $A_{ij}=-\Phi_2\lambda_p$ and $A_{ii}=f(\lambda_p,i)$, i,j taking the values 1, 3, 5, ..., ∞.

The determinant A_{ij} is convergent and all its diagonal elements are quadratic in λ_p. Hence the determinant has an infinite number of positive and negative roots. Due to condition (8), only the negative values of λ_p are admissible.

Using the method suggested by Singh [14], the values of λ_p and the coefficients $A_n^{(p)}$ are obtained numerically.

Hence θ is determined as

$$\theta=\sum_{p=1,3}^{\infty}2a_p\sum_{n=1,3}^{\infty}A_n^{(p)}\exp(\lambda_p\xi)\times\sin\frac{n\pi}{2}\eta+\bar{A}\xi+F(\eta), \quad (17)$$

where the constant a_p is calculated numerically with the help of the boundary condition (2).

The mean mixed temperature θ_m and the local Nusselt number Nu are defined as

$$\theta_m=\int_{-1}^{1}\theta u\,d\eta\Big/\int_{-1}^{1}u\,d\eta \quad (18)$$

and

$$Nu=\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=1}\Big/(\theta_m-\theta_w), \quad (19)$$

where

$$\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=1}=1.$$

This completes the mathematical analysis of the problem.

NUMERICAL RESULTS AND DISCUSSIONS

To analyse the effects of the Hall and the ion slip currents on the heat-transfer coefficients, the constants a_p, $A_n^{(p)}$ and λ_p in equation (17) have been calculated numerically for a set of representative values of the parameters. Table 1 gives the numerical values of the first five eigenvalues λ_ps. Using these numerical values of the constants, the temperature θ and the local Nusselt number Nu are obtained for $\beta_e=0.0$, 2.0; $\beta_i=0.0$, 0.5; $e_y=0.5$, 2.0; $Ha=40$, $Pe=500$ and $Ec\cdot Pr=1.0$ Figs. 1 and 2 give the development of temperature profiles $\theta-\theta_w$ and Nu.

As is clear from equation (17), the lowest eigenvalue, i.e. the smallest numerical value of λ_p, determines the entry length. This length is inversely proportional to λ_p. As noted from Table 1, due to the Hall and the ion slip currents, the magnitude of the lowest eigenvalue increases, leading to the conclusion that the thermal entry length is reduced due to these secondary currents.

The development of temperature profiles $\theta-\theta_w$ are plotted against η for different values of ξ in Fig. 1. The subscript I indicates the case $\beta_e=0.0=\beta_i$, subscript II indicates $\beta_e=2$, $\beta_i=0$, subscript III indicates $\beta_e=2$, $\beta_i=0.5$. It is seen that near the channel entrance, i.e. for small values of ξ,

$$(\theta-\theta_w)_{\rm I}<(\theta-\theta_w)_{\rm III}<(\theta-\theta_w)_{\rm II}$$

and as the temperature approaches the fully developed value,

$$(\theta-\theta_w)_{\rm III}<(\theta-\theta_w)_{\rm I}<(\theta-\theta_w)_{\rm II}$$

for the generator mode ($e_y=0.5$). But in the accelerator mode, ($e_y=2.0$),

$$(\theta-\theta_w)_{\rm III}<(\theta-\theta_w)_{\rm II}<(\theta-\theta_w)_{\rm I}$$

for all values of ξ. This is true for large values of $Ec\cdot Pr$ (>0.5). For small values of $Ec\cdot Pr$ (<0.005), $(\theta-\theta_w)$ becomes negative for all these cases showing that there is a competitive action between the external loss of heat and the internal heat generation as shown in [3, 4] also.

The variations of the local Nusselt number Nu are presented in Fig. 2. The Nusselt number curves for the case I are higher than II and III near the entrance of the channel, i.e. $Nu_{\rm I}>Nu_{\rm III}>Nu_{\rm II}$. But, as ξ increases, $Nu_{\rm I}$ becomes smaller than $Nu_{\rm III}$ and remains greater than $Nu_{\rm II}$, i.e. $Nu_{\rm III}>Nu_{\rm I}>Nu_{\rm II}$ for the generator mode ($e_y=0.5$) and large values of $Ec\cdot Pr$ (>0.5). For accelerator mode ($e_y=2.0$), $Nu_{\rm III}>Nu_{\rm II}>Nu_{\rm I}$ for all values of ξ. This shows that the heat-transfer rates from the fluid are increased for the accelerator mode in

Table 1. Eigenvalues λ_ps for $Ha = 40$, $Pe = 500$ and $Ec \cdot Pr = 1.0$

p	$\beta_e = 0.0 = \beta_i$	$\beta_e = 2.0$, $\beta_i = 0.0$	$\beta_e = 2.0$, $\beta_i = 0.5$
1	3.6182	4.2329	5.0240
2	32.7667	37.1896	44.3226
3	88.0118	103.616	108.015
4	183.767	193.192	199.898
5	302.720	399.657	547.800

the presence of the Hall and the ion slip currents. A similar phenomenon is seen for the case of uniform wall temperature. In the generator mode, the rate of heat transfer is less with Hall and ion slip currents initially, but the pattern changes as the temperature approaches to that of the fully developed state whereas with uniform wall temperature [9], it was found that the heat-transfer rates are reduced.

CONCLUSIONS

Thus it is concluded that the Hall and the ion slip currents significantly affect the heat-transfer coefficients. Comparing the two cases, i.e. only the Hall currents and the Hall and the ion slip currents, it is found that the heat transfer from the fluid is less with Hall currents alone for any value of ξ.

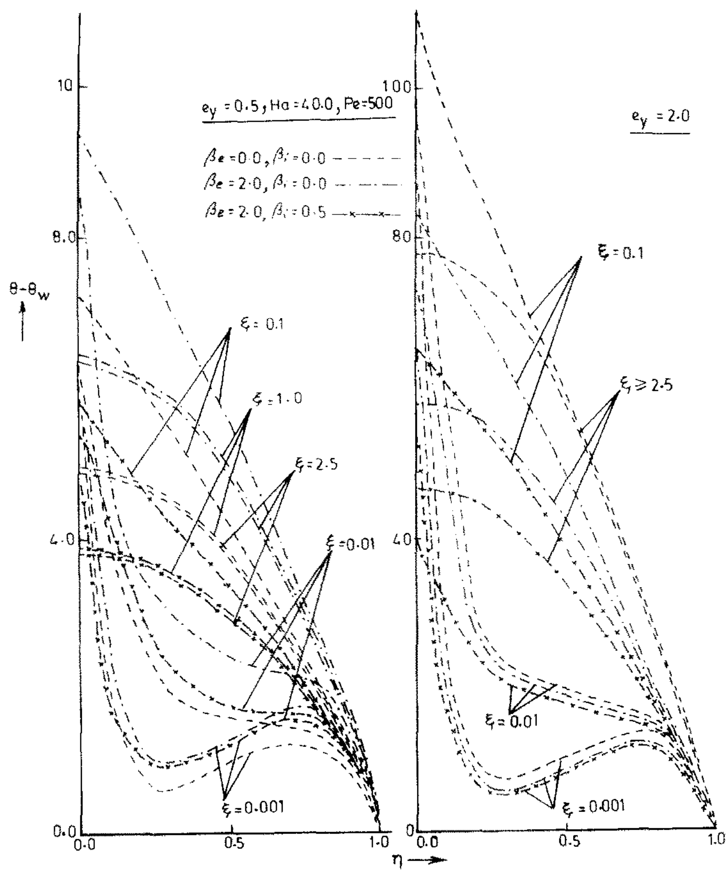

FIG. 1. Temperature distribution $\theta - \theta_w$ in the channel.

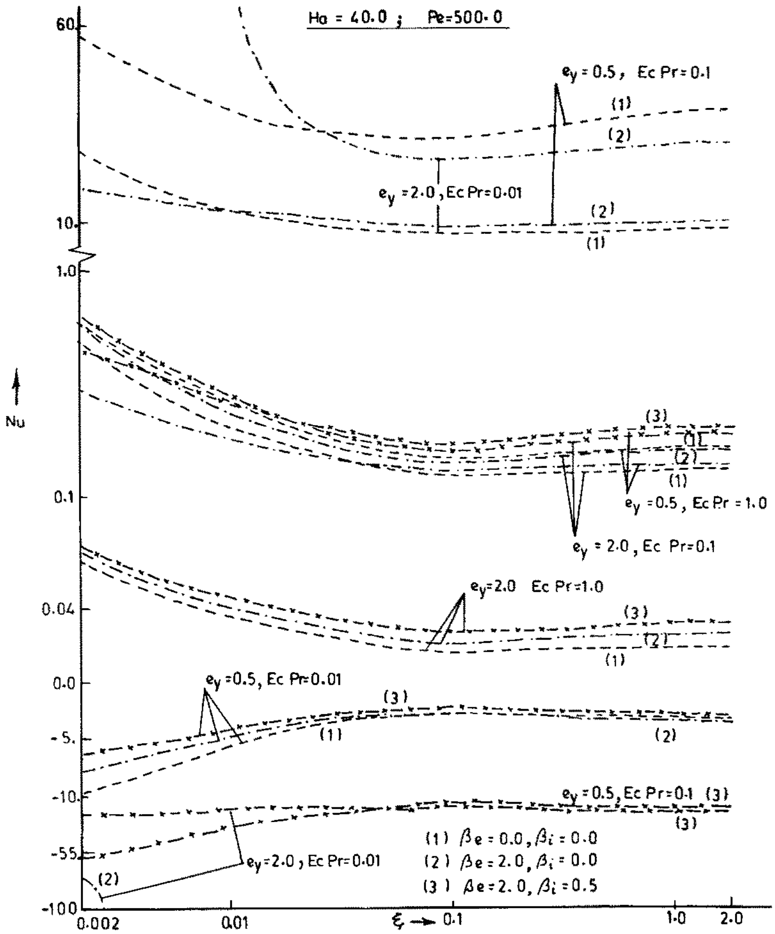

FIG. 2. Variation of Nusselt number Nu.

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APPENDIX

Expressions for the Constants in the Velocity and Temperature Distribution

$$a = [\tfrac{1}{2}\{(\varepsilon_1^2 + \varepsilon_2^2)^{1/2} + \varepsilon_1\}]^{1/2}$$
$$b = [\tfrac{1}{2}\{(\varepsilon_1^2 + \varepsilon_2^2)^{1/2} - \varepsilon_1\}]^{1/2}$$
$$\varepsilon_1 = \frac{Ha^2\alpha_e}{\alpha_e^2 + \beta_e^2}$$
$$\varepsilon_2 = \frac{Ha^2\beta_e}{\alpha_e^2 + \beta_e^2}$$
$$\varepsilon_3 = Re\frac{\partial P}{\partial \xi} - \frac{Ha^2}{\alpha_e^2 + \beta_e^2}(e_x\beta_e + e_y\alpha_e)$$
$$\varepsilon_4 = Re\frac{\partial P}{\partial \zeta} + \frac{Ha^2}{\alpha_e^2 + \beta_e^2}(e_y\beta_e - e_x\alpha_e)$$
$$C_{11} = -\frac{\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4}{\varepsilon_1^2 + \varepsilon_2^2}$$
$$D_{11} = -\frac{\varepsilon_4\varepsilon_1 - \varepsilon_3\varepsilon_2}{\varepsilon_1^2 + \varepsilon_2^2}$$
$$A_{11} = -\frac{D_{11}\sinh a\sin b + C_{11}\cosh a\cos b}{\cosh^2 a\cos^2 b + \sinh^2 a\sin^2 b}$$
$$B_{11} = \frac{C_{11}\sinh a\sin b - D_{11}\cosh a\cos b}{\cosh^2 a\cos^2 b + \sin^2 a\sin^2 b}.$$

Here $\partial P/\partial\xi$ and $\partial P/\partial\zeta$ are constants and their values are obtained from the following conditions:

(i) $\int_0^1 u\,d\eta = 1$, normalization condition for axial velocity profile.

(ii) $\int_0^1 v\,d\eta = 0$, there is no net mass flow cross-wise.

(iii) $\int_0^1 j_x\,d\eta = 0$, there is no net current flow in the axial direction.

Hence these conditions give

$$\frac{\partial P}{\partial \xi} = \frac{1}{Re}\left[\varepsilon_3 + \frac{Ha^2}{\alpha_e^2 + \beta_e^2}(e_y\alpha_e + e_x\beta_e)\right]$$
$$\frac{\partial P}{\partial \zeta} = \frac{1}{Re}\left[\varepsilon_4 + \frac{Ha^2}{\alpha_e^2 + \beta_e^2}(e_x\alpha_e - e_y\beta_e)\right]$$
$$e_x = (e_y^{-1})\beta_e/\alpha_e$$

where

$$\varepsilon_3 = \frac{1}{DEN}[\varepsilon_1\{a\sinh a\cosh a + b\cos b\sin b - (a^2 + b^2)(\sinh^2 a + \cos^2 b)\} - \varepsilon_2(b\sinh a\cosh a - a\sin b\cos b)](a^2 + b^2)(\sinh^2 a + \cos^2 b)$$

$$\varepsilon_4 = \frac{1}{DEN}[\varepsilon_1\{b\sinh a\cosh a - a\sin b\cos b\} + \varepsilon_2\{a\sinh a\cosh a + b\cos b\sin b - (a^2 + b^2)(\sinh^2 a + \cos^2 b)\}](a^2 + b^2)(\sinh^2 a + \cos^2 b)$$

$$DEN = (b\cosh a\sinh a - a\cos b\sin b)^2 + \{a\sinh a\cosh a + b\cos b\sin b - (a^2 + b^2)(\sinh^2 a + \cos^2 b)\}^2$$

$$N_1 = 0.5\,Ec\cdot Pr(a^2 + b^2)(A_{11}^2 + B_{11}^2)$$
$$N_2 = Ec\cdot Pr\cdot Ha^2\frac{\alpha_e}{\alpha_e^2 + \beta_e^2}[(e_x + D_{11})^2 + (e_y - C_{11})^2]$$
$$N_3 = Ec\cdot Pr\cdot Ha^2\frac{\alpha_e}{\alpha_e^2 + \beta_e^2}[(e_x + D_{11})B_{11} - (e_y - C_{11})A_{11}]$$
$$N_4 = Ec\cdot Pr\cdot Ha^2\frac{\alpha_e}{\alpha_e^2 + \beta_e^2}[(e_x + D_{11})A_{11} + (e_y - C_{11})B_{11}]$$

$$N_5 = 0.5\,Ec \cdot Pr \cdot Ha^2 \frac{\alpha_e}{\alpha_e^2 + \beta_e^2}[A_{11}^2 + B_{11}^2]$$

$$\bar{A} = 1 + Q_1 \sinh 2a + Q_2 \sin 2b + Q_3 + Q_4 \sinh a \cos b + Q_5 \cosh a \sin b$$

where

$$Q_1 = \frac{N_1 + N_5}{2a}$$

$$Q_2 = \frac{N_5 - N_1}{2b}$$

$$Q_3 = N_2$$

$$Q_4 = \frac{N_3 a - N_4 b}{a^2 + b^2}$$

$$Q_5 = \frac{N_3 b + N_4 a}{a^2 + b^2}$$

$$P_1 = -Q_1/2a$$

$$P_2 = Q_2/2b$$

$$P_3 = \bar{A} \cdot \frac{A_{11} 2ab - B_{11}(a^2 - b^2)}{(a^2 + b^2)^2} - \frac{N_3 2ab + N_4(a^2 - b^2)}{(a^2 + b^2)^2}$$

$$P_4 = \bar{A} \cdot \frac{(a^2 + b^2)A_{11} + B_{11} 2ab}{(a^2 + b^2)^2} - \frac{N_3(a^2 - b^2) - N_4 2ab}{(a^2 + b^2)^2}$$

$$P_5 = (\bar{A} C_{11} - N_2)/2$$

$$P_6 = -(P_1 \cosh 2a + P_2 \cos 2b + P_3 \sinh a \sin b + P_4 \cosh a \cos b + P_5)$$

$$\begin{aligned}
\Phi_1 &= \int_{-1}^{1} u \sin^2 \frac{n\pi}{2} \eta \, d\eta \\
&= \int_{-1}^{1} (A_{11} \cosh a\eta \cos b\eta - B_{11} \sinh a\eta \sin b\eta + C_{11}) \sin^2 \frac{n\pi}{2} \eta \, d\eta \\
&= \frac{A_{11}}{2} 2 \frac{a \sinh a \cos b + b \cosh a \sin b}{(a^2 + b^2)} \\
&\quad - \frac{a \sinh a \cos(n\pi + b) + (n\pi + b) \cosh a \sin(n\pi + b)}{a^2 + (n\pi + b)^2} \\
&\quad - \frac{a \sinh a \cos(n\pi - b) + (n\pi - b) \cosh a \sin(n\pi - b)}{a^2 + (n\pi - b)^2} \\
&\quad - \frac{B_{11}}{2} 2 \frac{a \cosh a \sin b - b \sinh a \cos b}{a^2 + b^2} \\
&\quad - \frac{a \cosh a \sin(n\pi + b) - (n\pi + b) \sinh a \cos(n\pi + b)}{a^2 + (n\pi + b)^2} \\
&\quad + \frac{a \cosh a \sin(n\pi - b) - (n\pi - b) \sinh a \cos(n\pi - b)}{a^2 + (n\pi - b)^2} + C_{11}
\end{aligned}$$

and

$$\Phi_2 = \int_{-1}^{1} u \sin \frac{n\pi}{2} \eta \sin \frac{m\pi}{2} \eta \, d\eta$$

$$= \int_{-1}^{1} (A_{11} \cosh a\eta \cos b\eta - B_{11} \sinh a\eta \sin b\eta + C_{11}) \cdot \sin \frac{n\pi}{2} \sin \frac{m\pi}{2} \eta \, d\eta$$

$$= \frac{A_{11}}{2} \frac{a \sinh a \cos\left(\overline{m - n} \frac{\pi}{2} + b\right) + \left(\overline{m - n} \frac{\pi}{2} + b\right) \cosh a \sin\left(\overline{m - n} \frac{\pi}{2} + b\right)}{a^2 + \left(\overline{m - n} \frac{\pi}{2} + b\right)^2}$$

$$
+\frac{a\sinh a\cos\left(\overline{m-n}\,\frac{\pi}{2}-b\right)+\left(\overline{m-n}\,\frac{\pi}{2}-b\right)\cosh a\sin\left(\overline{m-n}\,\frac{\pi}{2}-b\right)}{a^2+\left(\overline{m-n}\,\frac{\pi}{2}-b\right)^2}
$$

$$
-\frac{a\sinh a\cos\left(\overline{m+n}\,\frac{\pi}{2}+b\right)+\left(\overline{m+n}\,\frac{\pi}{2}+b\right)\cosh a\sin\left(\overline{m+n}\,\frac{\pi}{2}+b\right)}{a^2+\left(\overline{m+n}\,\frac{\pi}{2}+b\right)^2}
$$

$$
-\frac{a\sinh a\cos\left(\overline{m+n}\,\frac{\pi}{2}-b\right)+\left(\overline{m+n}\,\frac{\pi}{2}-b\right)\cosh a\sin\left(\overline{m+n}\,\frac{\pi}{2}-b\right)}{a^2+\left(\overline{m+n}\,\frac{\pi}{2}-b\right)^2}
$$

$$
-\frac{B_{11}}{2}\,\frac{a\cosh a\sin\left(\overline{m-n}\,\frac{\pi}{2}+b\right)-\left(\overline{m-n}\,\frac{\pi}{2}+b\right)\sinh a\cos\left(\overline{m-n}\,\frac{\pi}{2}+b\right)}{a^2+\left(\overline{m-n}\,\frac{\pi}{2}+b\right)^2}
$$

$$
-\frac{a\cosh a\sin\left(\overline{m-n}\,\frac{\pi}{2}-b\right)-\left(\overline{m-n}\,\frac{\pi}{2}-b\right)\sinh a\cos\left(\overline{m-n}\,\frac{\pi}{2}-b\right)}{a^2+\left(\overline{m-n}\,\frac{\pi}{2}-b\right)^2}
$$

$$
-\frac{a\cosh a\sin\left(\overline{m+n}\,\frac{\pi}{2}+b\right)-\left(\overline{m+n}\,\frac{\pi}{2}+b\right)\sinh a\cos\left(\overline{m+n}\,\frac{\pi}{2}+b\right)}{a^2+\left(\overline{m+n}\,\frac{\pi}{2}-b\right)^2}
$$

$$
+\frac{a\cosh a\sin\left(\overline{m+n}\,\frac{\pi}{2}-b\right)-\left(\overline{m+n}\,\frac{\pi}{2}-b\right)\sinh a\cos\left(\overline{m+n}\,\frac{\pi}{2}-b\right)}{a^2+\left(\overline{m+n}\,\frac{\pi}{2}-b\right)^2}.
$$

TRANSFERT DE CHALEUR DANS UN CANAL EN MHD AVEC FLUX THERMIQUE UNIFORME EN PAROI—EFFETS DES COURANTS HALL ET DE GLISSEMENT D'IONS

Résumé—Les effets de courants Hall et de glissement d'ions sur la convection forcée de chaleur dans la région d'entrée d'un canal magnétohydrodynamique ont été analysés en résolvant l'équation d'énergie sous l'angle d'un problème de valeur propres. On discute à la fois les modes de génération et d'accélération. Les transferts thermiques sont réduits par de tels courants.

WÄRMEÜBERTRAGUNG IN EINEM MHD-KANAL MIT GLEICHFÖRMIGEM WÄRMESTROM AN DER WAND—EINFLÜSSE DER HALL- UND DER IONENSLIP-STRÖME

Zusammenfassung—Die Einflüsse der Hall- und der Ionenslip-Ströme auf die erzwungene konvektive Wärmeübertragung im thermischen Einlaufgebiet eines magnetohydrodynamischen Kanals wurden durch Lösung der Energiegleichung als Eigenwert-Problem untersucht. Sowohl die Generator- als auch die Beschleuniger-Betriebsart werden diskutiert. Es wurde gefunden, daß diese Ströme eine Abnahme des Wärmeübergangs verursachen.

ТЕПЛОПЕРЕНОС В МАГНИТОГИДРОДИНАМИЧЕСКОМ КАНАЛЕ ПРИ РАВНОМЕРНОЙ ПЛОТНОСТИ ТЕПЛОВОГО ПОТОКА НА СТЕНКЕ. ВЛИЯНИЕ ТОКОВ ХОЛЛА И ИОННОГО СКОЛЬЖЕНИЯ

Аннотация — Влияние токов Холла и ионного скольжения на теплоперенос при вынужденной конвекции в тепловой входной области магнитогидродинамического канала анализировалось путем решения уравнения энергии как задачи на собственные значения. Рассмотрены режимы генерации и ускорения. Обнаружено, что указанные токи снижают интенсивность переноса тепла.